Control of a Low-Earth orbit satellite

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Part 1 : AOCS architecture

The mission you work on is a Low-Earth orbit satellite. You have to design the nominal mode of the mission. Here are the requirements the project manager gave you :

- Attitude estimation accuracy needed : better than 0.03°
- Agility : maximal angular velocity of 0.2°/s and maximal angular acceleration of $7.5\ 10^{-4}°/s^2$ on every satellite axis
- Cost minimization
- Mass minimization
- Power consumption minimization

The inertia of the satellite is given as $I = \begin{bmatrix} 30 & 0 & 0 \\ 0 & 29 & 0 \\ 0 & 0 & 31 \end{bmatrix} kg.m^2$

Part of this data is taken from [1].

Below are the available sensors and actuators :

Sensor	Accuracy (°)	Mass (kg)	Consumption (W)	Cost
Earth sensor	0.03	3.4	7.5	$
Sun sensor	0.18	0.23	0	$
Star tracker	0.008	3.7	5	$$

Actuators	Max angular momentum (Nms)	Maximal torque (Nm)	Mass (kg)	Consumption (W)	Cost
Magnetotorquer	infinite	0.00001	2	5	$
Reaction wheel	0.11	0.0005	5	10	$$
Control moment gyro	0.12	0.08	10	15	$$$

1) Find a combination of sensors/actuators that can meet your requirements for this mode of operation. Justify your choices.

2) You now have to design a control law for this mode of operation of the satellite. To do so, you need to define your system.
   a. Justify the fact that you can design one control law for each satellite axis (X, Y and Z).
   b. Write the equation of the dynamics of the satellite along the X-axis in the satellite frame. We assume the actuators and sensors are perfect.
   c. Write this equation in the Laplace domain assuming the angular velocity is small.
   d. Analyse the stability and the poles of the open-loop system

3) Choice of the type of controller :
   a. Prove that the system cannot be stabilized with a proportional gain
   b. Can the system be stabilized with a proportional-derivative controller ?
   Is the control law robust to a constant disturbance torque? Justify your answer.
   c. Prove that the system can be stabilised with a PID controller and be robust to a constant the disturbance torque.

4) Design of the controller

The controller needs to be robust to the uncertainties on the system :

- +/- 30 % on the inertia value
- +/-10% of torque error created by the reaction wheel
- Delays in the control loop :
  - Sensor delay : 300ms
  - On-board computer delay : 200ms
  - Actuator delay : 100ms
  - Discretization of the control law : 4Hz

a. Deduce the requirements on the gain and delay margins. We assume a phase margin requirement constraint of 45° (typical) and a required closed-loop bandwith of 0.01Hz.
b. Design a controller meeting these requirements.
c. Check and indicate the stability margins and the closed-loop bandwith of the system
d. Write the poles of the system and their damping ratios. Check the closed-loop temporal response.

Part 2 : Control loop simulation

1) Kinematics and dynamics equations
    a. In order to develop the simulator, write the kinematics equations relating the attitude of the satellite and the angular velocity. Use the quaternions formulation.
    b. Include in the dynamic equation derived in Part 1 question2b the reaction wheel.

2) Simulink simulator : Develop a Simulink simulator using the functions given in the LMS and including your control law. Add the reaction wheel modelling as a first-order system with a cut-off frequency of 1 rad/s. Include in your model the calculation of the reaction wheel angular momentum.

3) Check the temporal response of your system with a step of 0.001°. Identify the modes that you had Part 1 question 4d in the output attitude.

4) Verify the robustness of your system by simulating these conditions :
    a. With an inertia of 39kg.m² and 43kg.m²
    b. With an added delay of 850ms and 1s
    c. Does your control law comply with the requirements ?

5) Disturbance rejection
    a. With a null target angle, include this disturbance model, typical in low-Earth orbit [1]:
    $$T_d = T_0 + T_1 sin(\omega_0 t + \phi_1) + T_2 \sin(2\omega_0 t \ + \ \phi_2)$$
    With $T_0 = 10^{-6}$Nm, $T_1 = 3.1\ 10^{-5}$Nm, $T_2 = 1.5\ 10^{-5}$Nm and $\omega_0 = 0.001$ rad/s the orbital frequency.
    b. Plot and explain the output of your system subject to only this disturbance.
    c. Plot and explain the angular momentum values of the reaction wheels. If the simulation goes on for a long time, what will happen to the reaction wheel ? What will be the impact on the control law ? What have you planned in your AOCS architecture to mitigate this effect ?

Part 3 : Control of the flexible mode

You have just been informed that the solar panel of the satellite may have an impact on the AOCS stability and the mechanics engineer wants your opinion for the choice of their natural frequency. Two options are currently on the table :
- Natural frequency of 0.008Hz
- Natural frequency of 2Hz

In order to give your insight, a study of the impact of the flexible modes is necessary.

The model of a flexible mode is given as :

$$C_s = \frac{L^2 s^2}{s^2 + ds + K}\ddot{\theta}$$

With θ the attitude, C_s the torque created by the solar panels, $d = 0.005$ the damping ratio, $L = 2$ ($\boldsymbol{kg}.\boldsymbol{m}$²)1/2 the modal participation of the mode and K the pulsation of the mode.

1) Write the new dynamics equation of the satellite in the Laplace domain

2) Include the modelling of the flexible mode in your simulator. Simulate a target attitude change of 0.001° and check the temporal response of the system for the two natural frequencies. Is the system robust ? Are the performances degraded ?

3) The flexible mode needs to be taken into account in the design of the controller. Assuming that there are no external disturbances, the new dynamic equation reads :
$$C_c + \ C_s = I\ddot{\theta}$$

With C_c the torque command.

a. Write the transfer function between the attitude θ and the command C_c.
b. Add to your previously tuned controller a phase-lead controller of the type :

$$H(s) = \frac{1 + aTs}{1 + Ts}$$

Tune the a and T parameters for both natural frequencies
Can the system be stabilized in both cases? Is the temporal response affected by the flexible mode ?
c. Instead of a phase-lead controller, you now want to test a control of the flexible mode by gain. The requirement is to have a gain of the flexible mode <-5dB in closed-loop to limit its perturbation. What controller can you add to your PID ?
d. Check the margins and the closed-loop bandwith for both natural frequencies cases. Plot the temporal response in both cases.

4) From your analysis, what combination of natural frequency/controller type would you recommend ?

[1] « Demeter: A Benchmark For Robust Analysis And Control Of The Attitude Of Flexible Micro Satellites », C. Pittet and D. Arzelier, *5th IFAC Symposium on Robust Control Design*, 2006